New Journal of Physics
An Institute of Physics and Deutsche Physikalische Gesellschaft Journal

Bright matter wave solitons in Bose–Einstein condensates

K E Strecker[1], **G B Partridge**[1], **A G Truscott**[2] **and**
R G Hulet[1,3]
[1] Department of Physics and Astronomy and Rice Quantum Institute,
Rice University, Houston, TX 77251, USA
[2] Research School of Physical Sciences and Engineering, Australian National
University, ACT 0200, Australia
E-mail: randy@atomcool.rice.edu

New Journal of Physics **5** (2003) 73.1–73.8 (http://www.njp.org/)
Received 25 March 2003
Published 16 June 2003

Abstract. We review recent experimental and theoretical work on the creation of bright matter wave solitons in Bose–Einstein condensates. In two recent experiments, solitons are formed from Bose–Einstein condensates of ^{7}Li by utilizing a Feshbach resonance to switch from repulsive to attractive interactions. The solitons are made to propagate in a one-dimensional potential formed by a focused laser beam. For repulsive interactions, the wavepacket undergoes dispersive wavepacket spreading, while for attractive interactions, localized solitons are formed. In our experiment, a multi-soliton train containing up to ten solitons is observed to propagate without spreading for a duration of 2 s. Adjacent solitons are found to interact repulsively, in agreement with a calculation based on the nonlinear Schrödinger equation assuming that the soliton train is formed with an alternating phase structure. The origin of this phase structure is not entirely clear.

Contents

---

[3] Author to whom any correspondence should be addressed.

1367-2630/03/000073+8$30.00

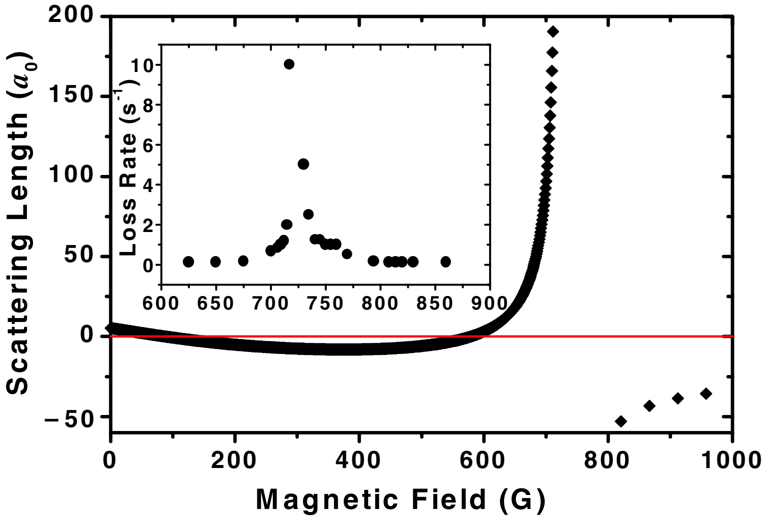

Figure 1. Feshbach resonance for the $(F, m_F) = (1, 1)$ state of ^{7}Li. The main graph is a calculation showing a resonance in the scattering length near 725 G, and a zero crossing near 550 G [13]. The inset shows the measured loss of atoms as a function of magnetic field. A sharp increase in the loss rate, characteristic of Feshbach resonances with bosons, is observed near the peak of the resonance.

1. Introduction

Although Bose–Einstein condensation (BEC) can occur in a gas of non-interacting particles, many of the properties of trapped atom condensates are a result of interparticle interaction. If the interactions are repulsive, the condensate is stable and its size and number have no fundamental limit. However, if the interactions are attractive, only a limited number of atoms can form a condensate [1], where in this case, the condensate is stabilized against collapse by the confining potential. Beyond a critical number of atoms, the condensate will collapse [2]–[4]. If the confining potential is made asymmetric, such that the atoms can only undergo one-dimensional (1D) motion, they are predicted to form a stable, self-focusing BEC or matter–wave soliton [5]–[7].

Solitons arise as a general solution to the nonlinear wave equation. They occur in many physical systems, including water waves, plasma waves, and light waves. Solitons are formed when the nonlinear term in the wave equation exactly compensates for wavepacket dispersion. A condensate can be described by the nonlinear Schrödinger equation, where the nonlinear term arises from the interatomic interactions [8]. The nonlinear term is cubic in the atomic field, in analogy with the well known Kerr nonlinearity in optics. For a condensate, the sign and magnitude of the nonlinearity are determined by the scattering length a. The interactions are repulsive for $a > 0$ and attractive for $a < 0$. Dark solitons have been seen in Bose–Einstein condensates with positive a [9]–[12]. A dark soliton is a localized *absence* of atoms or depression of the atomic field. Dark solitons are constrained to propagate in the nonlinear medium, which in this case is the condensate itself. Bright solitons, on the other hand, are themselves condensates and have no such constraint on the propagation medium.

Two groups, one at the Ecole Normale Supérieure in Paris [14] and our group at Rice University [15], have used a Feshbach resonance to create bright matter wave solitons in a BEC of ^{7}Li. A Feshbach resonance is a scattering resonance in which pairs of free atoms are tuned via the Zeeman effect into resonance with a vibrational state of the diatomic molecule [16]. This enables a to be continuously tuned from positive to negative values. A typical resonance, this one for ^{7}Li, is shown in figure 1. An experimental signature of a Feshbach resonance with bosons is an enhanced loss of trapped atoms [17] caused by an increased rate of inelastic collisions, such as molecule formation. A measurement of the losses for ^{7}Li is shown in the inset of figure 1. The Feshbach resonance provides a continuous knob to adjust the atom–atom interaction from repulsive to attractive, and from weak to strong.

Soliton stability is discussed in detail by Salasnich *et al* [18]. Essentially, a soliton must be strongly confined in all but one spatial dimension. Assuming harmonic confinement with cylindrical symmetry, stability requires that $N \lesssim \ell_r/|a|$, where N is the number of condensate atoms and $\ell_r = \sqrt{\hbar/m\omega_r}$ is the harmonic oscillator length corresponding to the radial confinement frequency ω_r. This condition ensures that radial excitations are 'frozen' out, and the motion of the soliton is purely 1D. Violation of this condition will lead to a mechanical collapse. In our previous experiments with condensates with attractive interactions [1]–[3], [19] the condensate was confined in a nearly spherical potential. In this case, stability is ensured only if excitations in *all* directions are frozen. This effectively corresponds to the '0D' limit. In both the Paris and Rice soliton experiments, a 1D confining potential is provided by the optical forces of a focused infrared laser beam.

2. Experiments

The Rice apparatus has been described generally in previous publications [15, 20]. We give a brief overview of the apparatus and experimental procedure here. The Paris experiment [14, 21] is very similar, but small differences between the experiments, which lead to somewhat different results, will be discussed later. Atoms from a thermal lithium beam are laser slowed and loaded into a magneto-optical trap. These atoms are then transferred to an electromagnetic Ioffe–Pritchard-type magnetic trap in the $F = 2, m_F = 2$ spin state and evaporatively cooled to near 1 μK. This state has a negative scattering length that is unaffected by magnetic field [22]. In order to take advantage of the Feshbach resonance shown in figure 1, however, the atoms must be in the $F = 1, m_F = 1$ state. Since this state is not magnetically trappable, the atoms are transferred to an optical trap consisting of a focused infrared Nd:YAG laser (1064 nm) for radial confinement, and two cylindrically focused doubled Nd:YAG beams (532 nm) 250 μm apart, providing 'end-caps' for axial confinement. After the magnetic trapping fields are switched off, a uniform magnetic field is ramped to 700 G, where the atoms are transferred to the $F = 1, m_F = 1$ state by an adiabatic microwave sweep. At this magnetic field, the scattering length has the large and positive value of $a \sim 200\ a_0$, where a_0 is the Bohr radius. The intensity of the infrared laser beam is then reduced by a factor of two. A large stable BEC forms as the gas cools by evaporation, facilitated by the large positive scattering length. Finally, the magnetic field is reduced to a value near 545 G, where a is small and negative. The atoms are detected by near-resonance imaging.

Soliton behaviour can be observed by setting the atoms in motion. This is achieved by repeating the experiment as described above, but with the infrared laser focus axially displaced from the centre of the magnetic trap and the end-caps. In this way, the BEC is initially formed on the side of the weak axial potential of the infrared laser beam and are held there by the end-caps. On removing the end-caps, the atoms oscillate in the axial potential for a varying length before being imaged. This process is repeated for different values of magnetic field and release time (figure 2).

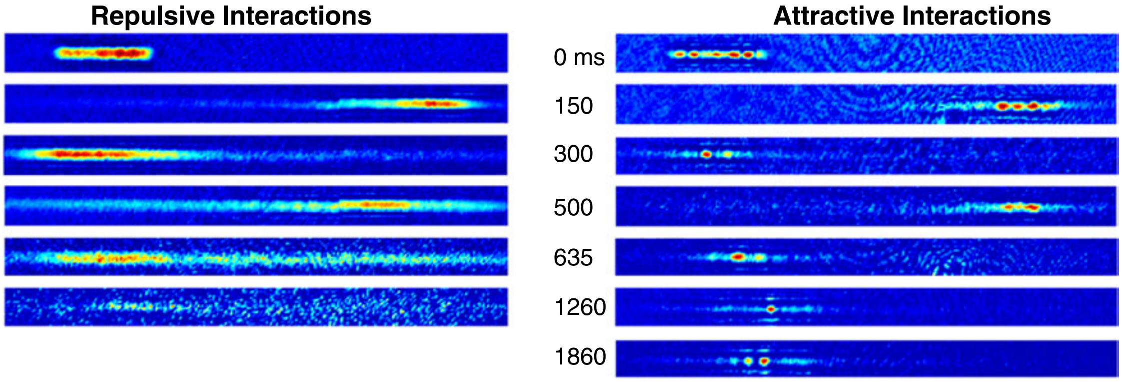

Figure 2. A comparison of the motion of a Bose–Einstein condensate with repulsive and attractive interactions under the influence of the harmonic axial potential. The axial frequency is approximately 3 Hz. The length of each frame corresponds to 1.28 mm in the plane of the atoms (reprinted from [15]).

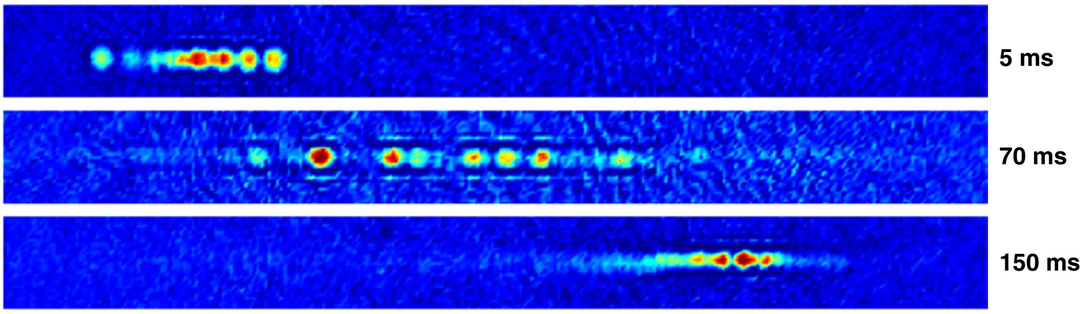

Figure 3. Soliton trains undergoing an oscillation in the axial potential. The solitons are seen to bunch at the turning points and spread out in the middle of the oscillation, suggesting repulsive soliton–soliton interactions. The number of observed solitons varies from shot to shot due to fluctuations in the initial number of atoms and a slow loss of signal with time (reprinted from [15]).

The images on the left are taken at a field of 630 G, where $a \sim +10\ a_0$, while the images on the right side are taken at a field of 545 G, corresponding to $a \sim -3\ a_0$. The condensate with positive a disperses as it propagates, while a condensate with negative a propagates without spreading, as expected for a soliton.

The images in figure 2 reveal that a soliton train is created, rather than a single soliton. More detailed images of the soliton train are shown in figure 3, where it can be seen that the solitons bunch at the turning points and spread out in the middle. From this we infer repulsive interactions between adjacent solitons.

3. Soliton formation and interactions

Theoretical modelling using the nonlinear Schrödinger equation shows that two solitons with a phase difference of π will interact repulsively [18, 23, 24], in agreement with the well known result first elucidated in the context of optical solitons in fibres [25]. This repulsion is a

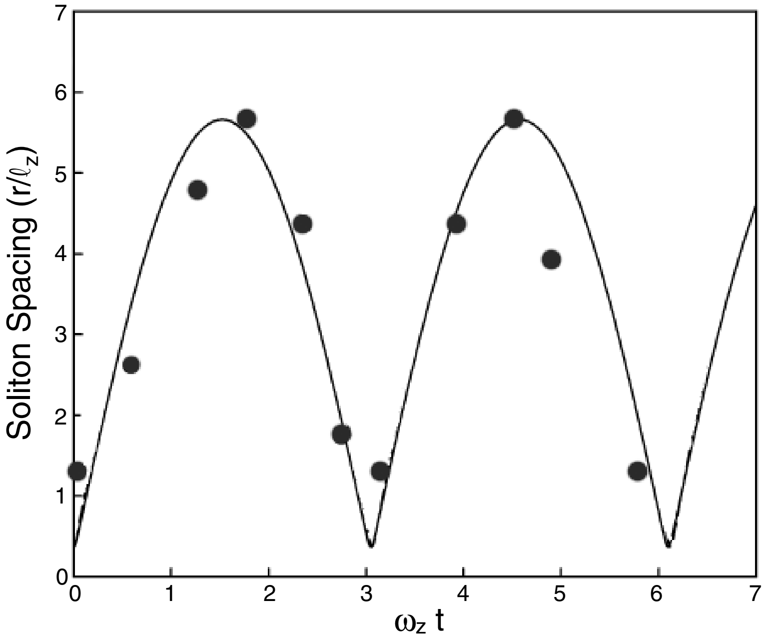

Figure 4. The relative spacing between solitons in the trap. The solid curve is a calculation of the equations of motion for two solitons in the trap using the nonlinear Schrödinger equation and assuming a phase difference of π between solitons [23]. The vertical axis is soliton separation (r) in units of the axial trap length, and the horizontal axis is time in units of the axial harmonic frequency times 2π. The solid points are data representing the separation of the brightest two adjacent solitons (reprinted from [23]).

manifestation of the wave nature of the solitons rather than the interatomic interactions, which are attractive. The relative motion decouples from the centre of mass motion, and the solitons are left to evolve in time. Figure 4 shows the theoretical results with a solid curve, while the data are shown by solid circles. The experiment and the theory are in good agreement. Salasnich *et al* [18] have assumed an alternating phase structure in a train of four solitons, and find good agreement with the observed dynamics [18].

An explanation of the formation of soliton trains is suggested by the presence of the alternating phase structure [15]. Upon changing the scattering length from positive to negative, the condensate becomes unstable to the growth of perturbations at a particular wavelength. The only available length scale is the condensate 'healing length', $\xi = (1/8\pi n|a|)^{1/2}$, where n is the atomic density. The healing length is also the characteristic length scale of a vortex in a superfluid. Initially the phase is constant across the condensate, but as the sign of the scattering length is switched, a mode with wavelength ξ becomes unstable, perhaps initiated by quantum mechanical fluctuations, and imprints the condensate with the alternating phase structure [23]. Although simplified, the numerical simulation was able to produce up to seven solitons with alternating phases.

The model suggests that the number of solitons produced should vary with the initial size of the condensate. To further test this theory, we attempted to change the initial size of the condensate before changing the sign of the scattering length. The same experimental procedure as above was followed except that the removal of the end-caps was delayed a time Δt before the sign of the scattering length was changed. This allows the condensate to expand for a time

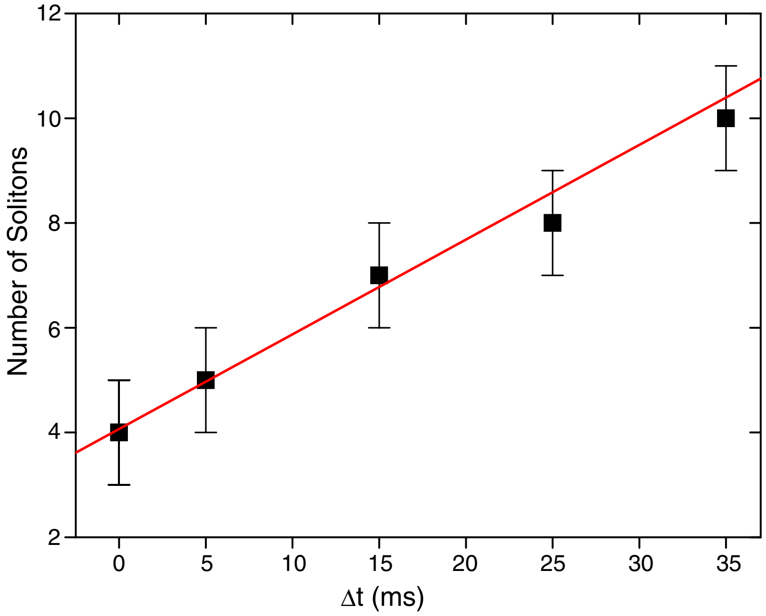

Figure 5. Number of solitons produced as a function of the release time from the end-caps, Δt. The data were recorded by varying Δt and waiting a fixed time before imaging the atoms. The error bars are due to uncertainty in identifying every soliton produced.

Δt before the solitons are formed. Figure 5 shows that the number of solitons formed increases linearly with Δt.

The mechanisms responsible for the formation of the soliton train and the long-term stability of solitons are not completely understood. A recent paper [26] suggests that solitons within a train are actually created with arbitrary phases. After a series of collapses induced by collisions between attracting solitons, a final stable configuration is achieved in which only repelling solitons remain. It is experimentally observed that the total number of atoms in the train ($<6 \times 10^4$) is a small fraction of the number of atoms in the initial, repulsive condensate (3×10^5). So although most atoms are lost to collapse, it is not clear whether they are lost predominantly in an initial collapse or to subsequent soliton collisions, as suggested in [26]. In either case, the loss occurs quickly compared to the relevant experimental timescale of 30 ms, which is the time required to change the magnetic field. The Paris and Rice soliton experiments are also similar to an experiment performed at JILA in ^{85}Rb, where a Feshbach resonance was used to suddenly switch the interactions from repulsive to attractive [4]. In that experiment, a small number of remnant atoms remained following the collapse of the condensate. In the Paris and Rice experiments, the condensate is confined to a 1D geometry, so the remnant atoms are solitons.

The Paris experiment produces only single solitons [14], rather than soliton trains. These contrasting results may be explained by two primary differences in the experiments:

(1) the axial potential was anti-trapping in the Paris experiment and

(2) the number of atoms in the initial condensate was an order of magnitude fewer than in the Rice experiment.

The anti-trapping potential causes the solitons to be accelerated out of the observation region in approximately 8 ms. This timescale is similar to the time for the condensate to collapse, so it is perhaps not surprising that the resulting products of the collapse are qualitatively different. The Paris group has shown that the mean square axial size of the soliton does not vary during its motion, while for condensates produced with $a = 0$ the size increases quadratically in time, as expected.

The key to these experiments is the ability to tune the atom–atom interactions smoothly from positive to negative. This interaction 'knob' has opened a new door for studying BEC with attractive interactions. There could also be practical applications of atom solitons. An atom soliton laser may be useful for precision measurements applications, as the input source to atom interferometer-based inertial and rotational sensors.

Acknowledgments

The authors gratefully acknowledge support from the National Science Foundation, the Office of Naval Research, NASA, and the Welch Foundation.

References

[1] Bradley C C, Sackett C A and Hulet R G 1997 Bose–Einstein condensation of lithium: observation of limited condensate number *Phys. Rev. Lett.* **78** 985–9

[2] Sackett C A, Gerton J M, Welling M and Hulet R G 1999 Measurement of collective collapse in a Bose–Einstein condensate with attractive interactions *Phys. Rev. Lett.* **82** 876–9

[3] Gerton J M, Strekalov D, Prodan I and Hulet R G 2000 Direct observation of growth and collapse of a Bose–Einstein condensate with attractive interactions *Nature* **408** 692–5

[4] Donley E A, Claussen N R, Cornish S L, Roberts J L, Cornell E A and Wieman C E 2001 Dynamics of collapsing and exploding Bose–Einstein condensates *Nature* **412** 295–9

[5] Ruprecht P A, Holland M J and Burnett K 1995 Time-dependent solution of the nonlinear Schrödinger equation for Bose-condensed trapped neutral atoms *Phys. Rev.* A **51** 4704–11

[6] Reinhardt W P and Clark C W 1997 Soliton dynamics in the collisions of Bose–Einstein condensates: an analog of the Josephson effect *J. Phys. B: At. Mol. Opt. Phys.* **30** L785–9

[7] Pérez-García V, Michinel H and Herrero H 1998 Bose–Einstein solitons in highly asymmetric traps *Phys. Rev.* A **57** 3837–42

[8] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 Theory of Bose–Einstein condensation in trapped gases *Rev. Mod. Phys.* **71** 463–512

[9] Burger S, Bongs K, Dettmer S, Ertmer W and Sengstock K 1999 Dark solitons in Bose–Einstein condensates *Phys. Rev. Lett.* **83** 5198–201

[10] Denschlag J *et al* 2000 Generating solitons by phase engineering of a Bose–Einstein condensate *Science* **287** 97–100

[11] Anderson B P, Haljan P C, Regal C A, Feder D L, Collins L A, Clark C W and Cornell E A 2001 Watching dark solitons decay into vortex rings in a Bose–Einstein condensate *Phys. Rev. Lett.* **83** 5198–201

[12] Dutton Z, Budde M, Slowe C and Hau L V 2001 Observation of quantum shock waves created with ultra-compressed slow light pulses in a Bose–Einstein condensate *Science* **293** 663–6

[13] McAlexander W I 2000 Collisional interactions in an ultracold lithium gas *PhD Thesis* Rice University

[14] Khaykovich L, Schreck F, Ferrari G, Bourdel T, Cubizolles J, Carr L D, Castin Y and Salomon C 2002 Formation of a matter-wave bright soliton *Science* **296** 1290–3

[15] Strecker K E, Partridge G B, Truscott A G and Hulet R G 2002 Formation and propagation of matter-wave soliton trains *Nature* **417** 150–3

[16] Tiesinga E, Verhaar B J and Stoof H T C 1993 Threshold and resonance phenomena in ultracold ground-state collisions *Phys. Rev.* A **47** 4114–22

[17] Inouye S, Andrews M R, Stenger J, Miesner H-J, Stamper-Kurn D M and Ketterle W 1998 Observation of Feshbach resonances in a Bose–Einstein condensate *Nature* **392** 151–4

[18] Salasnich L, Parola A and Reatto L 2002 Condensate bright solitons under transverse confinement *Phys. Rev.* A **66** 043603

[19] Bradley C C, Sackett C A, Tollett J J and Hulet R G 1995 Evidence of Bose–Einstein condensation in an atomic gas with attractive interactions *Phys. Rev. Lett.* **75** 1687–90

[20] Truscott A G, Strecker K E, McAlexander W I, Partridge G B and Hulet R G 2001 Observation of Fermi pressure in a gas of trapped atoms *Science* **291** 2570–2

[21] Schreck F, Ferrari G, Corwin K L, Cubizolles J, Khaykovich L, Mewes M-O and Salomon C 2001 Sympathetic cooling of bosonic and fermionic lithium gases towards quantum degeneracy *Phys. Rev.* A **64** 011402

[22] Abraham E R I, McAlexander W I, Gerton J M, Hulet R G, Côté R and Dalgarno A 1997 Triplet s-wave resonance in ^{6}Li collisions and scattering lengths of ^{6}Li and ^{7}Li *Phys. Rev.* A **55** R3299–302

[23] Al Khawaja U, Stoof H T C, Hulet R G, Strecker K E and Partridge G B 2002 Bright soliton trains of trapped Bose–Einstein condensate *Phys. Rev. Lett.* **89** 200404

[24] Leung V Y F, Truscott A G and Baldwin K G H 2002 Nonlinear atom optics with bright matter-wave soliton trains *Phys. Rev.* A **66** 061602

[25] Gordon J P 1983 Interaction forces among solitons in optical fibers *Opt. Lett.* **8** 596–8

[26] Carr L D and Brand J 2003 Spontaneous soliton formation and modulational instability in Bose–Einstein condensates *Preprint* cond-mat/0303257