Dissociation of one-dimensional matter-wave breathers due to quantum many-body effects

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We demonstrate that dissociation of one-dimensional cold-atom breathers, created by a quench from a fundamental soliton, is a quantum many-body effect, as all mean-field (MF) contributions to the dissociation vanish due to the integrability of the underlying nonlinear Schrödinger equation. The analysis predicts a possibility to observe quantum many-body effects without leaving the MF range of experimental parameters. In particular, the dissociation time on the order of a few seconds is expected for a typical atomic-soliton setting.

Under normal conditions, interacting quantum Bose gases do not readily exhibit signatures of their corpuscular nature, remaining thus effectively indistinct from their mean-field (MF) counterparts. Observability of microscopic effects involving substantial part of particles in a macroscopic system requires generally a beyond-MF density range, *viz.*, low density in 1D [1, 2] and high density in 3D. In 3D, the high density Lee-Huang-Yang corrections were realized experimentally using Feshabach resonance[3] and (in dipolar gases) [4–6] in the form of "quantum droplets", i.e. self-trapped states in bosonic gases which are stabilized against the collapse by the beyond-MF self-repulsion predicted theoretically in [7–9]. In particular, this scheme may lend stability to a binary gas with strong attraction between the components, which is necessary for the formation of the "droplets". Quantum effects involving a macroscopic number of atoms in collapsing attractive 3D gases and colliding condensates were also observed[10–12] and analyzed[13, 14] in the MF density range.

A generic opportunity to observe beyond-MF effects arises when some nontrivial conservation laws, that the MF dynamics obeys, prohibit a particular effect, but become broken at the microscopic level. In particular, the scale invariance governing the dynamics of a harmonically trapped 2D Bose gas cancels the interaction-induced shift of the frequency of monopole excitations, for all excitation amplitudes; however, this scale invariance is broken by the full quantum many-body Hamiltonian, leading to a small, albeit discernable on a zero background, shift [15]. The symmetry breaking by the secondary quantization may be considered as a manifestation of the general phenomenon known as the quantum anomaly [16]. In this Letter we develop a similar strategy for observing beyond-MF effects in the 1D attractive Bose gas, for a standard MF set of parameters. In this case, the MF equation amounts to the nonlinear Schrödinger (NLS) equation, which is integrable via the Inverse Scattering Transform (IST) [17]. The IST rigidly links the structure of a time-dependent solution to its initial form, with many features of the latter rendered identifiable in the former. In particular, a quench in the form of sudden increase of the strength of the attractive coupling constant by a factor of 4, converts a fundamental soliton into a superposition of two solitons with mass ratio 3:1, zero relative velocity, zero sepration between the solitons, and zero background [18, 19]. The two overlapping solitons have different chemical potentials, hence the density oscillates as a result of the quantum interference between the solitons. Such a superposition of solitons is usually identified as an NLS breather, by analogy with breathers appearing as solutions of the sine-Gordon equation (in this connection, it has been recently suggested that quantum many-body suppression of density oscillations in a one-dimensional bosonic breather occurs even in the MF regime proper [20]).

Quantum one- and two-soliton states have also been studied [21–25]. In particular, it transpired that in a full quantum many-body theory—contrary to its MF counterpart—the COM position of a soliton is a quantum coordinate whose conjugate velocity is subject to quantum fluctuations [26]. Below, we suggest that the spread of the relative velocity of the two solitons, produced from the original fundamental soliton by the fourfold quench of the coupling constant can be observable experimentally as a many-body quantum effect, thanks to the absence of any classical source of the relative velocity, as the MF breather generated by the quench does not split.

Note that while the physical object that we study—the NLS breather—is similar to the one considered in a recent manuscript [20] (with the Lieb-Liniger model replaced by a Hubbard lattice under the thermalization hypothesis), the effect addressed there, *viz.*, the quantum decay of the breathing, is clearly a different objective.

We consider N atoms of mass m moving in the 1D direction (x), with zero-range inter-atomic interactions of strength $g < 0$. The corresponding Hamiltonian reading

[27]

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + g\sum_{j<j'}\delta(x_j - x_{j'}) \ . \qquad (1)$$

This problem has an exact Bethe-ansatz solution [21, 28]. Due to the translational invariance of the Hamiltonian (1), its eigenfunctions have a homogeneous density. For attractive interactions with $g < 0$, they contain several strings – bound states of several atoms [22, 29]. A superposition of the eigenfunctions with different string velocities can be localized such that its density tends to the multi-soliton one in the limit $N \to \infty$ [23–25]. Normalization factors for multi-string states were obtained in [30]. We assume that at $t < 0$, the interaction strength was $g_0 = g/4$, and the system contained a single-string state $\varphi_N^{(0)}$ with zero center-of-mass (COM) velocity. After the application of the quench, $g_0 \to g$, the system's state will be a superposition of a single-string state φ_N, double-string states $\varphi_{N_1 N_2 v}$, where v is the relative velocity of two strings composed of N_1 and N_2 atoms, and multi-string states. A localized fundamental quantum soliton is a superposition of the single-string states with different COM velocities. States with different COM velocity are decoupled due to its conservation, therefore probabilities of transitions due to the quench from the fundamental soliton state to multi-soliton states will be the same as for the delocalized string states. The probabilities are calculated analytically using the exact Bethe-ansatz solution. The probability to remain in the single-string state can be explicitly calculated as

$$\left|\left\langle \varphi_N^{(0)} | \varphi_N \right\rangle\right|^2 = \left(\frac{2\sqrt{|g g_0|}}{|g| + |g_0|}\right)^{2(N-1)} = \left(\frac{4}{5}\right)^{2(N-1)}$$

For the double-string states the probabilities depend on the relative string velocity v and the string composition,

$$\frac{dP_{N_1 N_2}(v)}{dv} = \frac{N_1 N_2}{N}\left|\left\langle \varphi_N^{(0)} | \varphi_{N_1 N_2 v} \right\rangle\right|^2 \ . \qquad (2)$$

Examples of the probabilities are presented in Fig. 1. The natural velocity scale is

$$v_0 = |g|/(2\hbar) \qquad (3)$$

Total probabilities of the transition to double-string states

$$P_{N_1} = (2 - \delta_{N_1 N/2})\int_0^\infty \frac{dP_{N_1 N-N_1}(v)}{dv}dv \qquad (4)$$

are presented in Fig. 2. The cumulative probability of the transition to all double-string states is about 0.85 for $N \geq 8$. The transition $N \to 3N/4 + N/4$ features the *largest probability*, in agreement with the mean-field predictions. Probability distributions for the relative velocity of the dissociation products, averaged over dissociation channels,

$$P(v) = \sum_{N_1=1}^{N/2}(2 - \delta_{N_1 N/2})\frac{dP_{N_1 N-N_1}(v)}{dv}, \qquad (5)$$

is almost independent of N being plotted as a function of the ratio $v/\sqrt{N}$ (see Fig. 3). The numerically calculated half-width half-maximum (HWHM) Δv of the velocity distribution defined by $P(\Delta v) = P(0)/2$, can be fitted to

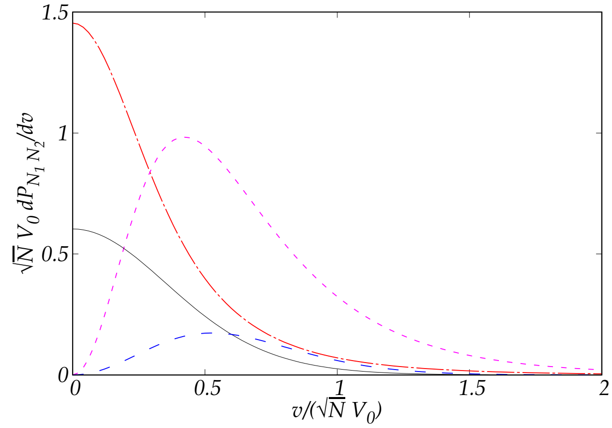

FIG. 1. Channel-selective probability distributions for the relative velocity [see Eq. (2)] of the dissociation products, produced by the application of the quench to the single string (fundamental quantum soliton). The black solid and red dot-dashed lines show $dP_{15,5}/dv$ and $dP_{3,1}/dv$, respectively, corresponding to the mean-field prediction, while $10^5 dP_{10,10}/dv$ and $10 dP_{2,2}/dv$ are shown by the blue long dashes and magenta short dashes, respectively. The velocity scale v_0 is defined by Eq. (3).

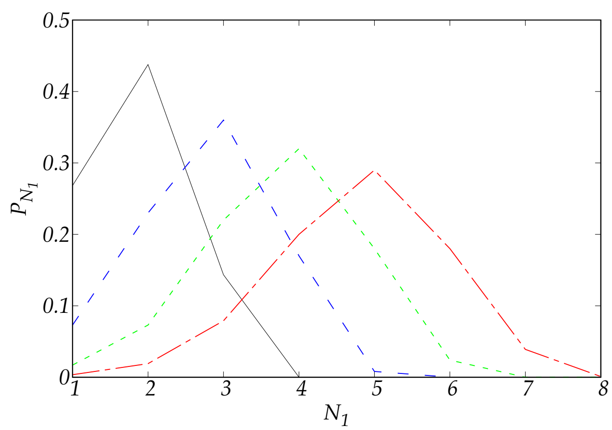

FIG. 2. Total probabilities for different dissociation channels (4), produced by the application of the $g/4 \to g$ quench to the single string (fundamental quantum soliton) composed of $N = 8$, 12, 16, 20 atoms (black solid, blue long-dashed, green short-dashed, and red dot-dashed lines, respectively).

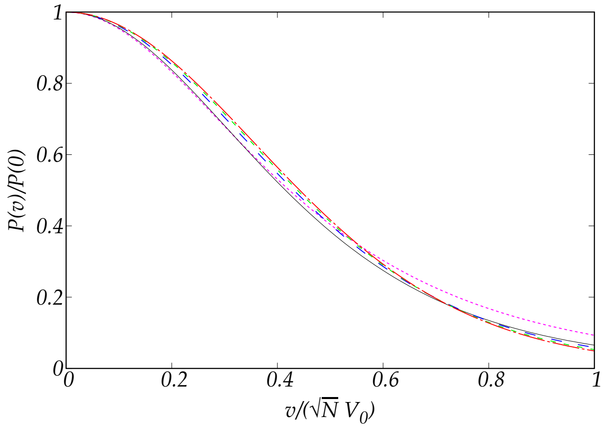

FIG. 3. Probability distributions for the relative velocity, averaged over dissociation channels [see Eq. (5)] of the dissociation products, produced by the application of the quench to the single string (fundamental quantum soliton) composed of $N = 4, 8, 12, 16, 20$ atoms (magenta dotted, black solid, blue long-dashed, green short-dashed, and red dot-dashed lines, respectively). The velocity scale v_0 is defined by Eq. (3).

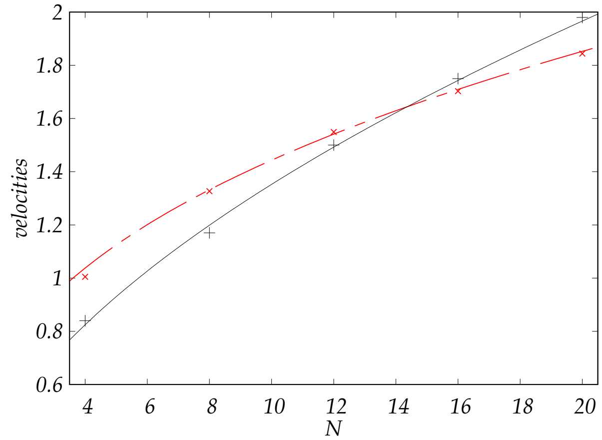

FIG. 4. HWHM (pluses) and r.m.s. (crosses) values of the relative velocity averaged over all two-string (two-soliton) dissociation channels, as a function of the number of atoms, N. The fits of Eqs. (6) and (7) are shown by the black solid and red dot-dashed lines, respectively. The velocity unit is v_0 is [see Eq. (3)].

the following law:

$$\Delta v \approx 0.39 N^{0.54} v_0, \tag{6}$$

see Fig. 4. The relative velocity can be measured also by its mean-squared value

$$\langle v^2 \rangle = \int_0^\infty v^2 P(v) dv \Big/ \int_0^\infty P(v) dv$$

However, the numerically found root-mean-square (r.m.s.) velocity increases with N only as

$$\sqrt{\langle v^2 \rangle} \approx 0.63 N^{0.36} v_0, \tag{7}$$

according to the fit displayed in Fig. 4. We conjecture that both measures of the relative velocity variation assume the same asymptotic scaling at very large N, the former law being closer to that asymptotic behavior. Indeed, the probability distribution (2) has slowly decaying tails for small N, e.g. $dP_{3N/4,N/4}(v)/dv \sim v^{-3N}$ at $v \to \infty$. The tails increase the r.m.s. velocity at small N. Thus, since the HWHM of the v distribution is evidently less connected to the tails than its r.m.s. counterpart, it is expected to converge to its large-N limit faster.

The following estimate confirms the $\sqrt{N}$ scaling for a typical relative velocity of the solitons, δv. Consider the system placed in an external harmonic-oscillator (HO) potential with frequency Ω. Varying Ω from vanishingly small values towards very large ones, at each value one can apply the $g/4 \to g$ quench to the respective ground state. The figure of merit to monitor is $\delta\tilde{x}$—the time-averaged distance, further symmetrized over permutations, between COMs of two groups of atoms, each containing the number of atoms $\sim N$. At small Ω, the state obtained right after the quench is unaffected by the external confinement, hence the two solitons (strings) start their motion with the free-space relative velocity δv. Thus, the distance $\delta\tilde{x}$ will be dominated by the typical distance between the solitons placed in the HO potential, $\delta v/\Omega$, which diverges at small Ω. This very long scale governs the estimate for $\delta\tilde{x}$, the other potentially relevant length scale, the distance between two atoms inside the same soliton, which is on the order of the size of an individual soliton, $\sim \hbar^2/m|g|N$, does not diverge at $\Omega \to 0$. Thus,

$$\delta\tilde{x}_{\Omega\to 0} \sim \frac{\delta v}{\Omega} \,.$$

On the other hand, at large Ω, the effect of the interatomic interactions vanishes and the estimate for $\delta\tilde{x}$ is determined by zero-point quantum fluctuations of the COM position of the cloud containing $\sim N$ particles:

$$\delta\tilde{x}_{\Omega\to\infty} \sim \sqrt{\frac{\hbar}{Nm\Omega}} \,.$$

A crossover between the two regimes occurs when the interaction energy per particle (comparable to the chemical potential of the gas, μ), $\sim \mu \sim mg^2N^2/\hbar^2$, becomes comparable to the HO quantum, $\hbar\Omega$. Indeed, when the former is dominated over by the latter, the interactions are irrelevant, and the system becomes an HO-confined ideal gas. At the crossover, the two above estimates yield the same value. An estimate for δv immediately follows:

$$\delta\tilde{x}_{\Omega\to 0}|_{\mu\sim\hbar\Omega} \sim \delta\tilde{x}_{\Omega\to\infty}|_{\mu\sim\hbar\Omega} \;\Rightarrow$$
$$\delta v \sim \sqrt{\frac{\hbar\Omega}{Nm}}\Big|_{\Omega\sim\frac{mg^2N^2}{\hbar^3}} \sim \frac{|g|}{\hbar}\sqrt{N} \,.$$

Indeed, this estimate is consistent with the fit (6).

The above results suggest that experimental observation of the variance in the relative velocity of the solitons due to quantum many-body effects may be possible. To demonstrate this, we consider 3×10^3 ^{7}Li atoms, in a waveguide with a transverse trapping frequency $\omega_{\rm perp} = 2\pi \times 254$ Hz. The initial state is a fundamental matter-wave soliton, existing at scattering length $a_{t<0} = -1\, a_{\rm Bohr}$, which is quenched up to $a_{t>0} = -4\, a_{\rm Bohr}$. The resulting state constitutes an NLS breather with an aphelion density profile proportional to $\mathrm{sech}^2(x/\ell_{\rm breather})$ and width $\ell_{\rm breather} = 8\hbar^2/(mgN) = 36.\ \mu$m [18, 19] [31]. Assuming that the splitting of the breather into two solitons becomes apparent when the distance between their COMs, after evolution time τ, $\Delta x = \Delta v \tau$ becomes comparable to the breather width $\ell_{\rm breather}$, and using extrapolation (6) for the relative velocity of the solitons, we obtain $\tau = 3$ s for the time necessary to certainly observe the splitting of the breather.

The predicted dissociation time can be made even shorter at the expense of reducing the cloud population. Indeed, assuming that the scattering length is increases accordingly in such a way that the product Na is kept at a finite fraction of the collapse critical value $Na \lesssim a_{\rm perp}$, $a_{\rm perp} \sim \sqrt{\hbar/(m\omega_{\rm perp})}$ being the size of the transverse vibrational ground state of the waveguide used. The microscopic velocity scale v_0, the separation velocity Δv, and the breather size $\ell_{\rm breather}$ can be estimated as $v_0 \lesssim \hbar/(ma_{\rm perp}N)$, $\Delta v \lesssim \hbar/(ma_{\rm perp}\sqrt{N})$, and $\ell_{\rm breather} \gtrsim a_{\rm perp}$ respectively. Then the breather dissociation time diminishes as $\tau \sim \ell_{\rm breather}/\Delta v \gtrsim (1/\omega_{\rm perp})\sqrt{N}$ as the number of particles decreases.

To summarize, we have showed that dissociation of the 1D matter-wave breather created by the quench from a fundamental soliton is a purely quantum many-body effect, as all the MF contributions to the dissociation vanish due to the integrability at the MF level. This conclusion opens the way to observe the truly quantum many-body effect without leaving the MF range of experimental parameters. We have evaluated the dissociation time corresponding to typical atomic-soliton experiments.

In further work on this topic, special attention will have to be paid to the role of the real-world deviations—usually neglected—from the idealized 1D model with zero noise in both the two-body interaction constant and the single-body potential energy. Some clues can be gained from attempts to experimentally observe the quantum deviation from the scale-invariance-induced constancy of the monopole frequency of the 2D Bose gas. In that context, quantum many-body effects are masked by phenomena caused by weak dependence of the quantum state on the third, confined dimension [32].

An example which makes it possible to explicitly compare the MF approximation and its many-body counterpart in the 3D geometry is offered by the problem of the stabilization of the gas of bosons with repulsive interactions, attracted to the center with potential $\sim -r^{-2}$. In that case, the MF predicts suppression of the quantum collapse and creation of a ground state which is missing in the single-particle case [33], while the full many-body analysis demonstrates that the same newly created state exists as a metastable one [34].

Such effects, along with other distortions of the idealized model [35–37] constitute a subject of the future research.

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[1] T. Kinoshita, T. Wenger, and D. S. Weiss, Science **305**, 1125 (2004).
[2] B. L. Tolra, K. M. O'Hara, J. H. Huckans, W. D. Phillips, S. L. Rolston, and J. V. Porto, Phys. Rev. Lett. **92**, 190401 (2004).
[3] N. Navon, S. Piatecki, K. Günter, B. Rem, T. C. Nguyen, F. Chevy, W. Krauth, and C. Salomon, Phys. Rev. Lett. **107**, 135301 (2011).
[4] I. Ferrier-Barbut, H. Kadau, M. Schmitt, M. Wenzel, and T. Pfau, Phys. Rev. Lett. **116**, 215301 (2016).
[5] M. Schmitt, M. Wenzel, F. Boettcher, I. Ferrier-Barbut, and T. Pfau, Nature **539**, 259 (2016).
[6] L. Chomaz, S. Baier, D. Petter, M. J. Mark, F. Wächtler, L. Santos, and F. Ferlaino, Phys. Rev. X **6**, 041039 (2016).
[7] D. S. Petrov, Phys. Rev. Lett. **115**, 155302 (2015).
[8] D. S. Petrov and G. E. Astrakharchik, Phys. Rev. Lett. **117**, 100401 (2016).
[9] D. Baillie, R. M. Wilson, R. N. Bisset, and P. B. Blakie, Phys. Rev. A **94**, 021602 (2016).
[10] E. Donley, N. Claussen, S. Cornish, J. Roberts, E. Cornell, and C. Wieman, Nature **412**, 295 (2001).
[11] J. M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. **89**, 020401 (2002).
[12] J. K. Chin, J. M. Vogels, and W. Ketterle, Phys. Rev. Lett. **90**, 160405 (2003).
[13] V. A. Yurovsky, Phys. Rev. A **65**, 033605 (2002).
[14] V. I. Yukalov and E. R. Yukalova, Laser Physics Letters **1**, 50 (2004).
[15] M. Olshanii, H. Perrin, and V. Lorent, Phys. Rev. Lett. **105**, 095302 (2010).
[16] K. S. Gupta and S. G. Rajeev, Phys. Rev. D **48**, 5940 (1993).
[17] M. J. Ablowitz and H. Segur, *Solitons and the Inverse Scattering Transform* (SIAM, Philadelphia, 1981).
[18] V. E. Zakharov and A. B. Shabat, Soviet Physics JETP **34**, 62 (1972).

[19] J. Satsuma and N. Yajima, Supp. Progr. Theor. Phys. **55**, 284 (1974).

[20] C. Weiss and L. D. Carr, "Higher-order quantum bright solitons in Bose-Einstein condensates show truly quantum emergent behavior," Preprint at arXiv:1612.05545 (2016).

[21] F. A. Berezin, G. P. Pohil, and V. M. Finkelberg, Vestnik Moskovskogo Universiteta (in Russian) **No. 1**, 21 (1964).

[22] J. B. McGuire, J. Math. Phys. **5**, 622 (1964).

[23] F. Calogero and A. Degasperis, Phys. Rev. A **11**, 265 (1975).

[24] Y. Lai and H. A. Haus, Phys. Rev. A **40**, 844 (1989).

[25] Y. Lai and H. A. Haus, Phys. Rev. A **40**, 854 (1989).

[26] Y. Castin, Eur. Phys. J. B **68**, 317 (2009).

[27] V. A. Yurovsky, M. Olshanii, and D. S. Weiss, in *Adv. At. Mol. Opt. Phys.*, Vol. 55 (Elsevier Academic, New York, 2008) p. 61.

[28] E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963).

[29] C. N. Yang, Phys. Rev. **168**, 1920 (1968).

[30] P. Calabrese and J.-S. Caux, Phys. Rev. Lett. **98**, 150403 (2007).

[31] Here and below, we are using the "deep 1D" expression $g = 2\hbar a \omega_{\mathrm{perp}}$ [38] for the effective coupling constant. For the present set of parameters, the number of atoms is only two times lower than the collapse critical number, $N_{\mathrm{crit.}} = 7900$, hence deviations from the purely 1D behavior are expected.

[32] K. Merloti, R. Dubessy, L. Longchambon, A. Perrin, P.-E. Pottie, V. Lorent, and H. Perrin, New Journal of Physics **15**, 033007 (2013).

[33] H. Sakaguchi and B. A. Malomed, Phys. Rev. A **83**, 013607 (2011).

[34] G. E. Astrakharchik and B. A. Malomed, Phys. Rev. A **92**, 043632 (2015).

[35] Y. S. Kivshar and B. A. Malomed, Rev. Mod. Phys. **61**, 763 (1989).

[36] H. Sakaguchi and B. A. Malomed, Phys. Rev. E **70**, 066613 (2004).

[37] H. Yanay, L. Khaykovich, and B. A. Malomed, Chaos **19**, 033145 (2009).

[38] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998).