1D to 3D Crossover of a Spin-Imbalanced Fermi Gas

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(Dated: May 24, 2016)

We have characterized the one-dimensional (1D) to three-dimensional (3D) crossover of a two-component spin-imbalanced Fermi gas of $^6$Li atoms in a 2D optical lattice by varying the lattice tunneling and the interactions. The gas phase separates, and we detect the phase boundaries using in situ imaging of the inhomogeneous density profiles. The locations of the phases are inverted in 1D as compared to 3D, thus providing a clear signature of the crossover. By scaling the tunneling rate $t$ with respect to the pair binding energy $\epsilon_B$, we observe a collapse of the data to a universal crossover point at a scaled tunneling value of $t/\epsilon_B = 0.016(1)$.

PACS numbers: 67.85.Lm, 71.10.Pm, 37.10.Jk, 05.70.Fh

Spin-imbalanced atomic Fermi gases have been studied extensively in recent years, motivated by a search for exotic superfluid phases [1–3]. One such superfluid, the Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) phase [4, 5], has not been seen in three dimensions (3D) but is believed to occupy a large portion of the one-dimensional (1D) phase diagram [6, 7]. Measurements have confirmed that the 1D phase diagram is consistent with theories exhibiting FFLO [8], but no direct evidence for this phase has been obtained. The FFLO phase is expected to be more robust to quantum fluctuations in higher dimensions, however, thus focusing attention on the dimensional crossover [9–12].

Spin-imbalanced trapped Fermi gases have been observed to phase separate in both 3D [13–15] and in 1D [8]. The nature of this phase separation, is qualitatively different in 3D than in 1D, as shown schematically in Fig. 1(a). In 3D, phase separation of an interacting two-component spin-imbalanced Fermi gas results in a balanced superfluid SF0 core surrounded by polarized shells, while in 1D, the core is partially polarized. A crossover between the 1D and 3D regimes may be realized by varying the aspect ratio of the confining potential [16–20]. Although, a complementary dimensional crossover may be achieved by varying the tunneling $t$ between neighboring tubes aligned in an array, as depicted in Fig. 1(b). Such a geometry, which may be realized with an optical lattice is more analogous to some material systems, such as carbon nanotube bundles [21] and spin-$1/2$ magnet chains [22, 23]. The bundle will cross over from an array of independent 1D tubes for small $t$, to a 3D gas as $t$ is increased [24, 25]. We have employed this geometry to determine the crossover value of $t$ for various interaction strengths and find a striking universality in the crossover location.

As described in detail previously [8, 13], our experiment employs the lowest two hyperfine sublevels of $^6$Li, the $|F = 1/2, m_F = 1/2\rangle$ state, designated as $|\uparrow\rangle$, and the $|F = 1/2, m_F = -1/2\rangle$ state, designated as $|\downarrow\rangle$. These correspond to the majority and the minority states, respectively. The atoms are prepared in state $|\uparrow\rangle$, and the population imbalance is controlled by varying the power of a single frequency sweep through the $\sim 80$ MHz transition frequency between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states [8]. A final step of evaporative cooling is performed in an optical trap subsequent to the creation of the spin mixture [8]. A 2D optical lattice, as depicted in Fig. 1(b), is formed by an orthogonal pair of retro-reflected laser beams at a wavelength $\lambda$ of 1064 nm. The lattice depth $V_L$ may be controlled up to a maximum value of

![FIG. 1. (Color online) (a) Schematic of phase separation of a spin-imbalanced Fermi gas in 1D (top) and in 3D (bottom) at zero temperature. For 1D, the central region is an LO-type partially polarized superfluid (SF$_P$), while the wings are either a balanced superfluid (SF$_0$) for small central tube polarization $P$, or a fully polarized normal phase (N$_P$) (not shown) for $P$ above a critical value $P_{c1D}$. Below a critical polarization in 3D, $P_{c3D}$, a central SF$_0$ core is surrounded by a partially polarized superfluid (SF$_P$) or normal phase (N$_P$) depending on interactions, and finally, the outer shell is N$_P$.

(b) Schematic of an array of 1D coupled tubes formed by a 2D optical lattice. By decreasing the optical lattice depth, the tunneling rate $t$ between the tubes increases and the system crosses over from 1D to 3D.](image-url)
12 \ E_r \ using \ liquid \ crystal \ retarders \ (LCRs) \ to \ rotate \ the \ polarization \ of \ the \ retro-reflected \ beams \ with \ respect\ to \ the \ incoming \ beams. \ Here, \ E_r = \hbar^2k^2/2m \ is \ the \ lattice \ recoil \ energy, \ k = 2\pi/\lambda, \ and \ m \ is \ the \ atomic \ mass. \ The \ potential \ in \ the \ axial \ (z) \ direction \ is \ approximately \ harmonic \ with \ a \ frequency \ \omega_z \ and \ varies \ linearly \ with \ V_z \ from \ (2\pi)197 \ Hz \ for \ V_L = 2.5 \ E_r \ to \ (2\pi)256 \ Hz \ for \ V_z = 12 \ E_r. \ We \ find \ that \ the \ mean \ number \ of \ \langle |\uparrow\rangle \rangle \ atoms \ in \ the central \ tube, \ N_{\uparrow}, \ is \ between \ 160 \ and \ 240 \ for \ small \ (<5\%) \ polarizations, \ but \ it \ decreases \ for \ larger \ polarizations \ due \ to \ inefficient \ evaporation. \ The \ interaction \ strength \ between \ the \ two \ states \ is \ tuned \ via \ the \ wide \ Feshbach \ resonance \ located \ at \ 832.2 \ G \ \[26, \ 27\]. \ We \ define \ as \ \langle \langle |\uparrow\rangle \rangle \ \rangle \ \rangle,

It \ was \ previously \ shown \ that \ the \ axial \ cloud \ radii \ of \ the \ minority \ state \ distribution, \ R_{\uparrow}, \ and \ the \ spin \ difference \ distribution, \ R_d, \ determine \ the \ 1D-like \ phase \ boundaries \ \[8\]. \ These \ radii \ correspond \ to \ the \ axial \ location \ where \ the \ minority \ density \ and \ the \ spin \ difference \ density \ go \ to \ zero, \ respectively. \ Since \ the \ spin \ difference \ density \ is \ zero \ in \ the \ fully \ paired \ SF_0 \ phase, \ R_d \ corresponds \ to \ the \ boundary \ between \ the \ SF_\uparrow \ core \ and \ the \ SF_0 \ wings \ in \ 1D \ \(\text{Fig. 1(a)}\). \ R_d \ is \ zero \ for \ P_\uparrow = 0, \ but \ moves \ to \ larger \ axial \ radius \ with \ increasing \ P_\uparrow \ until \ the \ polarized \ core \ encompasses \ the \ entire \ cloud. \ At \ this \ polarization, \ which \ we \ define \ as \ \langle \langle P_\uparrow^{1D}\rangle \ \rangle \ \langle 6, \ 8\rangle, \ the \ entire \ tube \ is \ in \ the \ SF_\uparrow \ phase \ and \ R_d = R_{\uparrow}. \ At \ even \ larger \ P_\uparrow, \ the \ boundary \ between \ the \ SF_\uparrow \ core \ and \ the \ SF_0 \ wings \ is \ defined \ by \ R_{\uparrow}. \n
The \ radii \ R_\downarrow \ and \ R_d \ may \ be \ extracted \ from \ the \ full \ density \ distributions \ n(x, y, z) \ but \ they \ may \ also \ be \ obtained \ directly \ from \ the \ n_\uparrow(x, z) \ distributions \ by \ assuming \ the \ validity \ of \ the \ local \ density \ approximation \ (LDA) \ in \ the \ radial \ direction. \ In \ this \ case, \ the \ chemical \ potential \ of \ each \ spin \ state \ is \ maximized \ for \ the \ central \ tube \ so \ that \ the \ phase \ boundaries, \ R_{\uparrow} \ and \ R_d, \ are \ largest \ for \ the \ central \ tube \ and \ decrease \ radially. \ We \ therefore \ use \ the \ central \ axial \ cut \ \(x = 0\) \ of \ the \ n_\uparrow(x, z) \ distributions \ to \ locate \ R_{\uparrow} \ and \ R_d, \ as \ indicated \ in \ \text{Fig. 2(a)}. \n
There \ is \ always \ a \ shell \ structure \ in \ 3D \ \(\text{Fig. 1(a)}\) \ again, \ according \ to \ the \ LDA, \ either \ the \ density \ profiles \ or \ the \ column \ density \ profiles \ can \ be \ used \ to \ find \ the \ cloud \ radii, \ R_{\uparrow} \ and \ R_d, \ to \ locate \ the \ phase \ boundaries \ \[29, \ 30\]. \ At \ zero \ temperature, \ the \ center \ of \ the \ cloud \ is \ a \ balanced \ superfluid \ SF_0 \ for \ P_\uparrow \ less \ than \ a \ critical \ polarization \ \langle P_\uparrow^{1D}\rangle \ \langle 31\rangle, \ which \ is \ the \ Chandrasekhar-Clogston \ limit \ in \ the \ BCS \ regime \ \[32, \ 33\]. \ The \ radius \ where \ the \ spin-difference \ density \ first \ rises \ above \ zero \ from \ the \ center \ of \ the \ cloud \ determines \ the \ boundary \ between \ the \ SF_0 \ and \ the \ SF_\uparrow \ or \ SF_\downarrow \ \text{phases}. \ Outside \ the \ partially \ polarized \ region \ is \ a \ \text{SF}_0 \ shell, \ whose \ boundary \ is \ defined \ by \ R_{\downarrow}. \ Atoms \ in \ the \ \text{SF}_0 \ shell \ are \ non-interacting, \ so \ the \ outer \ boundary \ of \ the \ cloud \ is \ R_{\uparrow} \ which \ is \ also \ equivalent \ to \ R_d \ here. \n
\text{Figure 2(a) shows a 1D-like profile, where the spin-difference \ column \ density \ profile \ is \ approximately \ parabolic, \ in \ contrast \ to \ Fig. 2(b) \ where \ the \ nearly \ flat-topped \ profile \ indicates \ a \ central \ SF_0 \ phase \ that \ is \ consistent \ with \ 3D \ phase \ separation \ \[2, \ 34\]. \ The \ distinction \ between \ 3D \ and \ 1D \ phase \ separation \ is \ confirmed \ by \ examination \ of \ the \ local \ polarization \ \text{p}(0, 0, z) \ at \ the \ center \ of \ the \ cloud \ \text{p}_0 = \text{p}(0, 0, 0) \ obtained \ from \ the \ inverse \ Abel \ transformed \ data. \ In \ \text{Fig. 2(c)}, \ \text{p}_0 \ is \ greater \ than \ zero,}
implying a partially polarized phase consistent with
1D phase separation, while Fig. 2(d) shows an ex-
ample with $p_0 = 0$ and is therefore a 3D phase sepa-
ration containing an SF$_0$ core.

Two examples of phase diagrams constructed from
the radii $R_d$ and $R_i$ are presented in Figs. 3(a) and
(b). Figure 3(a) corresponds to a relatively deep latt-
ice, with $V_L = 12 E_r$, that exhibits a 1D-like phase
diagram for which the core is polarized. The distin-
guishing characteristics of the 1D-like phase diagram
are 1) $R_d$ crosses $R_i$ at a non-zero $P^1_D$, and 2) $R_d$
goes to zero as $P_t$ goes to zero. Figure 3(b) shows
an example of a 3D-like phase diagram where the
centrally located phase is SF$_0$. In a strictly 3D sys-
tem, $R_d$ decreases with decreasing $P_t$ until meeting
$R_i$ at $P_t = 0$. In this quasi-3D system, however, $R_d$
is seen to cross $R_i$ at non-zero $P_t$, corresponding to
a small, but non-zero $P^1_D$.

Phase separation in 3D is distinguished by the
presence of a superfluid core that is suppressed at a
critical polarization $P_c^{3D}$ [13–15]. The core radius
may be identified by locating the polarization where
$R_d$ first rises above zero from the center of the cloud.
Due to the noise in the inverse Abel transformed
data, however, we instead find $P^3D$ by measuring
$p_0$, which we find to be less sensitive to noise. $P^3D$
is the central tube polarization $P_t$, above which $p_0$
begins to rise from zero. For $P^3D = 0$, there is no
balanced core for any $P_t$, and thus the gas is 1D-like.
Figure 3(c) shows $p_0$ corresponding to the 1D phase
diagram of Fig. 3(a). In this case, $p_0$ increases lin-
early with $P_t$. By decreasing $V_L$ sufficiently, a kink
appears in $p_0$, as shown in Fig. 3(d), where the inter-
cept gives $P^1_D$. The insets of Fig. 3 show the fitted
portion of $p_0$ versus $P_t$ in detail.

![Figure 2](image1.png)

**FIG. 2.** (Color online) (a, b) Column density pro-
files and (c, d) the corresponding local polarization
$p(0,0,z)$ profiles of spin-imbalanced gases. These profiles
are smoothed using a Gaussian function with width
of 4 pixels, where 1 pixel $= 1.33 \mu m$. The values of mag-
etic field, lattice depth, and the central tube polarization
$P_t$ are indicated above each column. (a) and (b) The
$|↑\rangle$, $|↓\rangle$, and their difference are indicated by the
black, blue and red curves, respectively. $R_i$ and $R_d$ are
indicated by the red and blue vertical lines. (c) and (d)
Local polarization found from the difference between the
majority and minority density profiles along the central
tube using a weighted average of the central 7 tubes.
The green band shows the 13 $\mu$m region along $z$ that
is used to average the local polarization to find $p_0$. The
grayed out regions are where $N_i$ is consistent with the
background noise and thus, where the local polarization
is poorly defined. The entire cloud in (a) and (c) is SF$_0$, while in (b, d), there is an extended region of SF$_0$ in the
center of the cloud ($p_0 = 0$), then a region of SF$_P$ or
SF$_C$.

![Figure 3](image2.png)

**FIG. 3.** (Color online) (a) 1D- and (b) 3D-like phase
diagrams. $R_i$ (●) and $R_d$ (●) are scaled by $N^{1/2} L_z$ [6, 8],
where $L_z = \sqrt{\hbar / m \omega_z}$ is the axial harmonic oscillator
length and $N = N_t + N_i$. The colored regions corre-
spond to the indicated phases, as defined in Fig. 1. The
boundaries between the phases follow the data. In (b),
the open circle indicates the measured $P^3D$ from (d).
The dotted line is an extrapolation from $P^3D$. (c) and
(d) The local central polarization $p_0$ vs. $P_t$, used to find
$P^3D$. The insets show the central region near $P^3D$. The
solid red line is a fit to the data to find $P^3D$, using a
function with a bilinear slope [15]. The vertical arrows
indicate $P^1_D$ (purple) and $P^3D$ (green). The gas be-
comes more 3D-like as $P^3D$ decreases and $P^3D$ increases.
Each data point is the average of $\sim 10$ experimental re-
alizations, binned with width $\Delta P_t = 0.005$.

Figures 4(a) and (b) show $P^1D$ and $P^3D$ for sev-
eral interaction strengths. We calculate $t$ from the
eigenenergies of the 1D Hamiltonian [35] which in-
cludes nearest neighbor and next-nearest neighbor con-
tributions. The next-nearest neighbor term be-
gins to contribute at lattice depths below $5 E_r$. Fig-
FIG. 4. (Color online) (a) $P_c^{1D}$ and (b) $P_c^{3D}$ vs. $t$. Ordered from lowest to highest field, the corresponding $a_{3D}$ are: $6170 a_0$, unitarity, $-8610 a_0$, $-5360 a_0$, and $-4340 a_0$, in units of the Bohr radius $a_0$. The corresponding ranges of $\epsilon_B$ depending on lattice strength are: $3.8 - 5.2 E_r$, $2.5 - 3.7 E_r$, $1.9 - 2.9 E_r$, $1.6 - 2.5 E_r$, and $1.4 - 2.3 E_r$, respectively. (c) $P_c^{1D}$ and (d) $P_c^{3D}$ versus the scaled tunneling rate $\tilde{t} = t/\epsilon_B$. The critical polarizations collapse onto a single curve when plotted against the scaled tunneling $\tilde{t}$. The 1D to 3D crossover is indicated by the dotted line in (d), where $\tilde{t}_c = 0.016(1)$, the value above which the gas has an SF core. Fitting $P_c^{1D}$ in (b) to an exponential gives a $1/e$ decay of $\tilde{t} = 0.015(1)$. The uncertainty in $P_t$ is estimated from 10 images known to be balanced.

ures 4(a) and (b) show that the 3D regime is attained for larger $t$, as expected, but also for larger magnetic field, corresponding to weaker attractive interactions, and thus larger chemical potentials.

In Figs. 4(c) and (d), we replot the data against the scaled tunneling rate $\tilde{t} = t/\epsilon_B$, where $\epsilon_B$ is the pair binding energy, calculated from [36]

$$\frac{\sqrt{2} l_\perp}{a_{3D}} = -\zeta \left[ \frac{1}{2}, \frac{-\epsilon_B}{2\hbar \omega_\perp} \right],$$

and where $\zeta$ is the Hurwitz zeta function. This solution depends on the lattice frequency $\omega_\perp$, the transverse harmonic oscillator length $l_\perp = \sqrt{\hbar / m \omega_\perp}$, as well as the 3D s-wave scattering length $a_{3D}$. When scaled in this way the data collapse onto a single curve, thus demonstrating the universality of the crossover [9]. By fitting the data of Fig. 4(d) with $\tilde{t} < 0.04$ to a bilinear function, we find the intercept to be $\tilde{t}_c = 0.016(1)$. The uncertainty is the combination from the standard error of the fit and the uncertainty in the measurements of $V_L$ and the magnetic field. All clouds with $\tilde{t} > \tilde{t}_c$ possess a superfluid core, which is a 3D-like characteristic. As seen in Fig. 4(c), $P_c^{1D}$ decreases exponentially with a decay constant of $\tilde{t} = 0.015(1)$, where the uncertainty has the same origins. This suppression of $P_c^{1D}$ is quantitatively consistent with the onset of 3D behavior and provides a confirming measure of $\tilde{t}_c$.

A mean field analysis predicts that the phase boundary between the SF$_0$ core and the N$_{FP}$ phase corresponds to a first order transition [9]. Due to noise in the density profiles from the inverse-Abel transformed data, however, we are unable to directly observe a jump in the local polarization. This could also be a consequence of finite $T$. Finally, the mean-field analysis also predicts that the 3D to 1D crossover may be driven by increasing the chemical potential $\mu$ [9]. The slope of this boundary, however, is very steep in the $\mu$ vs. $h$ plane, where $h$ is the chemical potential difference. Since our measurements are performed in the regime where $P_t \to 0$, or equivalently $h \to 0$, a possible $\mu$-dependent transition could only occur at very large $\mu$ where the 1D...
criterion for each tube no longer holds.

Our results show that the 1D to 3D crossover occurs at a universal scaled tunneling, $t_c = 0.016(1)$. The crossover region is predicted to be the most robust against fluctuations in FFLO wavenumber and temperature [9], suggesting the most fruitful parameter region to search for the FFLO phase is the quasi-1D regime near $t_c$.

The authors would like to thank Erich Mueller, Dan Sheehy and David Huse for many valuable discussions. This work was supported by grants from the NSF, ONR, the Welch Foundation, and the ARO-MURI program.

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[31] Since $P_{c1D}$ is defined with respect to the central tube polarization, not the global cloud polarization, there is an offset in the polarization between our $P_{c1D}$ and previous determinations.